

# STAR FORMATION ON GALACTIC SCALES: EMPIRICAL LAWS

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**Abstract.** Empirical star formation laws from the last 20 years are reviewed with a comparison to simulations. The current form in main galaxy disks has a linear relationship between the star formation rate per unit area and the molecular cloud mass per unit area with a timescale for molecular gas conversion of about 2 Gyr. The local ratio of molecular mass to atomic mass scales nearly linearly with pressure, as determined from the weight of the gas layer in the galaxy. In the outer parts of galaxies and in dwarf irregular galaxies, the disk can be dominated by atomic hydrogen and the star formation rate per unit area becomes directly proportional to the total gas mass per unit area, with a consumption time of about 100 Gyr. The importance of a threshold for gravitational instabilities is not clear. Observations suggest such a threshold is not always important, while simulations generally show that it is. The threshold is difficult to evaluate because it is sensitive to magnetic and viscous forces, the presence of spiral waves and other local effects, and the equation of state.

## 1 Introduction: the Kennicutt-Schmidt law of Star Formation

Star formation and stellar evolution are such important drivers of galactic evolution that empirical laws to determine the star formation rate have been investigated for over 50 years. The results have never been very precise because star formation spans a wide range of scales, from cluster-forming cores to molecular clouds to the whole interstellar medium.

On the scale of a galaxy, the first idea was a proposed connection between the total star formation rate and the mass of interstellar gas. Schmidt (1959) derived the star formation rate (SFR) over the history of the Milky Way assuming a constant initial luminosity function for stars,  $\Psi(M_V)$ , a stellar lifetime function  $T(M_V)$ , a gas return per star equal to all of the stellar mass above  $0.7 M_\odot$ , and a

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star formation rate  $f(t)$  that scales with a power  $n$  of the gas mass,  $M_G(t)$ . Then  $f(t)\Sigma_{MV}\Psi(M_V) = C[M_G(t)]^n$ , for a summation  $\Sigma_{MV}$  over all stellar types.

Schmidt gave analytical solutions for  $n = 0, 1, 2$ . He noted that a scale height for HI of 144 pc, a scale height for Cepheids of 80 pc, and a scale height for clusters of 58 pc gave  $n = 2$  to 3. The white dwarf count gave  $n > 2$ , the He abundance suggested  $n = 2$ , the uniformity of HI suggested  $n \geq 2$ , and the cluster mass function gave  $n = 1$  to 2. Schmidt also suggested that with  $n = 2$ , dense galaxies like ellipticals should now have less gas than low-density galaxies like the LMC. His final comment was “It is hoped to study the evolution of galaxies in more detail in the future.” Following Schmidt (1959), many authors derived scaling relations between the average surface density of star formation,  $\Sigma_{SFR}$ , and the average surface density of gas. Buat, Deharveng & Donas (1989) included molecular and atomic gas and determined star formation rates from the UV flux corrected for Milky Way and internal extinction. They assumed a constant  $H_2/CO$  ratio and a Scalo (1986) IMF. The result was a good correlation between the average star formation rate in a sample of 28 galaxies and the  $1.65 \pm 0.16$  power of the average total gas surface density. In the same year, Kennicutt (1989) used  $H\alpha$  for star formation, and HI and CO for the gas with a constant  $H_2/CO$  conversion factor, and determined star formation rates both as a function of galactocentric radius and averaged over whole galaxy disks. For whole galaxies, the average  $H\alpha$  flux scaled with the average gas surface density to a power between 1 and 2; there was a lot of scatter in this relation and the correlation was better for HI than  $H_2$ . More interesting was Kennicutt’s (1989) result that the star formation rate had an abrupt cutoff in radius where the Toomre (1964) stability condition indicated the onset of gravitationally stable gas. Kennicutt derived a threshold gas column density for star formation,  $\Sigma_{crit} = \alpha\sigma\kappa/(3.36G)$  for  $\alpha = 0.7$ ;  $\sigma$  is the velocity dispersion of the gas;  $\kappa$  is the epicyclic frequency, and  $G$  is the gravitational constant.

In a second study, Kennicutt (1998) examined the disk-average star formation rates using a larger sample of galaxies with  $H\alpha$ , HI, and CO. He found that for normal galaxies, the slope of the SFR-surface density relation ranged between 1.3 to 2.5, depending on how the slope was measured; there was a lot of scatter. When starburst galaxies with molecular surface densities in excess of  $100 M_\odot$  were included, the overall slope became better defined and was around 1.4. This paper also found a good correlation with a star formation rate that scaled directly with the average surface density of gas and inversely with the rotation period of the disk. This second law suggested that large-scale dynamical processes are involved.

Hunter et al. (1998) considered the same type of analysis for dwarf Irregulars and derived a critical surface density that was lower than the Kennicutt (1989) value by a factor of  $\sim 2$ . This meant that stars form in more stable gas in dwarf irregulars compared to spirals.

Boissiet et al. (2003) compared  $\Sigma_{SFR}$  and  $\Sigma_{gas}$  versus radius in 16 resolved galaxies with three theoretical expressions. The best fits were a SFR dependence on the gas surface density as  $\Sigma_{SFR} \propto \Sigma_{gas}^{2.06}$ , a more dynamical law from Boissier & Prantzos (1999) which gave the fit  $\Sigma_{SFR} \propto \Sigma_{gas}^{1.48}(V/R)$  for rotation

speed  $V$  and radius  $R$ , and a third type of law from Dopita & Ryder (1994), which fit to  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{0.97} / \Sigma_{\text{tot}}^{0.61}$ . Boissiet et al. (2003) assumed that  $\text{H}_2/\text{CO}$  varied with radius as the metallicity (Boselli et al., 2002). Their conclusion was that the three laws are equally good, and that for the pure gas law,  $n > 1.4$ . Boissiet et al. (2003) also looked for a star formation threshold in the Milky Way. They determined  $\Sigma/\Sigma_{\text{crit}}$  using both pure-gas for  $\Sigma_{\text{crit}}$  and a gas+star  $\Sigma_{\text{crit}}$  from Wang & Silk (1994). They found that the gas+star  $\Sigma_{\text{crit}}$  gave the best threshold for determining where star formation occurs. The gas alone was sub-threshold throughout the disk.

Zasov & Smirnova (2005) showed that a threshold like  $\Sigma_{\text{crit}}$  may be used to determine the gas fraction in galaxies. If all galaxies have  $\Sigma(\text{HI})$  approximately at the critical  $\Sigma_{\text{crit}} = \alpha\kappa\sigma/\pi G$ , which is proportional to  $V/R$  from  $\kappa$ , then  $M_{\text{gas}} = \int_R 2\pi R \Sigma_{\text{crit}} dR \propto VR$ . This was shown to be the case from observations. They also considered that the total mass is  $M_{\text{tot}} \propto V^2 R$ , in which case  $M_{\text{tot}}/M_{\text{gas}} \propto V$ , the rotation speed. This was also shown to be confirmed by observations. In their interpretation, small galaxies are more gas-rich than large galaxies because all galaxies have their gas column densities close to the surface density threshold.

For the Milky Way, Misiriotis et al. (2006) used COBE/DIRBE observations to get both the gas and dust distributions and the SFR distribution. They found a gas-law slope of  $2.18 \pm 0.20$ , which they claimed was similar to Kennicutt's (1998) bivariate fit slope  $n = 2.5$  for normal galaxies. Luna et al. (2006) determined the Milky Way SFR from IRAS point sources and the CO surface density from a southern hemisphere survey (assuming constant  $\text{H}_2/\text{CO}$ ). They found star formation concentrated in low-shear spiral arms and suggested an additional dependence on shear. Overall they derived  $\Sigma_{\text{SFR}} \sim \Sigma_{\text{gas}}^{1.2 \pm 0.2}$ . Vorobyov (2003) also suggested a shear dependence for the SFR based on observations of the Cartwheel galaxy, where there is an inner ring of star formation with high shear that is too faint for the normal Kennicutt law, given the gas column density.

### 1.1 The $Q$ Threshold

A threshold for gravitational instabilities in rotating disks has been derived for various ideal cases. For an infinitely thin disk of isothermal gas, the dispersion relation for radial waves is  $\omega^2 = k^2\sigma^2 - 2\pi G\Sigma k + \kappa^2$ . Solving for the fastest growth rate  $\omega$  gives the wavenumber at peak growth,  $k = \pi G\Sigma/\sigma^2$ , and the wavelength,  $\lambda = 2\sigma^2/G\Sigma$ , which is on the order of a kiloparsec in main galaxy disks. The dominant unstable mass is  $M \sim (\lambda/2)^2\Sigma = \sigma^4/G^2\Sigma \sim 10^7 M_\odot$  in local spirals. The peak rate is given by

$$\omega_{\text{peak}}^2 = -(\pi G\Sigma/\sigma^2)^2 + \kappa^2 = -(\pi G\Sigma/\sigma^2)^2(1 - Q^2) \quad (1.1)$$

which requires  $Q \equiv \kappa\sigma/\pi G\Sigma < 1$  for instability (i.e., when  $\omega_{\text{peak}}^2 < 0$ ).

Disk thickness weakens the gravitational force in the in-plane direction by an amount that depends on wavenumber, approximately as  $1/(1+kH)$  for exponential scale height  $H$  (e.g., Elmegreen, 1987; Kim & Ostriker, 2007). Typically,  $k \sim 1/H$ , so this weakening can slow the instability by a factor of  $\sim 2$ , and it can make

the disk slightly more stable by a factor of 2 in  $Q$ . On the other hand, cooling during condensation decreases the effective value of the velocity dispersion, which should really be written  $\gamma^{1/2}\sigma$  for adiabatic index  $\gamma$  that appears in the relation  $\delta P \propto \delta\rho^\gamma$  with pressure  $P$  and density  $\rho$ . If  $P$  is nearly constant for changes in  $\rho$ , as often observed, then  $\gamma \sim 0$ . Myers (1978) found  $\gamma \sim 0.25$  for various thermal temperatures at interstellar densities between  $0.1 \text{ cm}^{-3}$  and  $100 \text{ cm}^{-3}$ . Thus the effects of disk thickness and a soft equation of state partially compensate for each other.

There is also a  $Q$  threshold for the collapse of an expanding shell of gas (Elmegreen, Palous & Ehlerova, 2002). Pressures from OB associations form giant shells of gas and cause them to expand. Eventually they go unstable when the accumulated gas is cold and massive enough, provided the induced rotation and shear from Coriolis forces are small. Considering thousands of initial conditions, these authors found that a sensitive indicator of whether collapse occurs before the shell disperses is the value of  $Q$  in the local galaxy disk, i.e., independent of the shell itself. The fraction  $f$  of shells that collapsed scaled inversely with  $Q$  as  $f \sim 0.5 - 0.4 \log_{10} Q$ .

The Toomre  $Q$  parameter is also likely to play a role in the occurrence of instabilities in turbulence-compressed gas on a galactic scale (Elmegreen, 2002). Isothermal compression has to include a mass comparable to the ambient Jeans mass,  $M_{\text{Jeans}}$ , in order to trigger instabilities. The turbulent outer scale in the galaxy is comparable to the Jeans length,  $L_{\text{Jeans}}$ , which is about the galactic gas scale height,  $H$ . If the compression distance exceeds the epicyclic length, then Coriolis forces spin up the compressed gas, leading to resistance from centrifugal forces. So instability needs  $L_{\text{Jeans}} \leq L_{\text{epicycle}}$ , which means  $Q \leq 1$ , since  $L_{\text{Jeans}} \sim H \sim \sigma^2/\pi G\Sigma$ . The epicyclic length is  $L_{\text{epicycle}} \sim \sigma/k$ , so  $L_{\text{Jeans}}/L_{\text{epicycle}} = Q$ .

The dimensionless parameter  $Q$  measures the ratio of the centrifugal force from the Coriolis spin-up of a condensing gas perturbation to the self-gravitational force, on the scale where gravity and pressure forces are equal, which is the Jeans length. The derivation of  $Q$  assumes that angular momentum is conserved, so the Coriolis force spins up the gas to the maximum possible extent. When  $Q > 1$ , a condensing perturbation on the scale of the Jeans length spins up so fast that its centrifugal force pulls it apart against self-gravity. Larger-scale perturbations have the same self-gravitational acceleration (which scales with  $\Sigma$ ) and stronger Coriolis acceleration (which scales with  $\kappa^2/k$ ); smaller-scale perturbations have stronger accelerations from pressure. If angular momentum is not conserved, then the disk can be unstable for a wider range of  $Q$  because there is less spin up during condensation. For example, the Coriolis force can be resisted by magnetic tension or viscosity and then the angular momentum in a condensing cloud will get stripped away. This removes the  $Q$  threshold completely (Chandrasekhar, 1954; Stephenson, 1961; Lynden-Bell, 1966; Hunter & Horak, 1983). In the magnetic case, the result is the Magneto-Jeans instability, which can dominate the gas condensation in low-shear environments like spiral arms and some inner disks (Elmegreen, 1987, 1991, 1994; Kim & Ostriker, 2001, 2002; Kim et al., 2002). For the viscous case, Gammie (1996) showed that for  $Q$  close to but larger than 1, i.e.,

in the otherwise stable regime, viscosity can make the gas unstable with a growth rate equal to nearly one-third of the full rate for a normally unstable ( $Q < 1$ ) disk. A dimensionless parameter for viscosity  $\nu$  is  $\nu\kappa^3/G^2\Sigma^2$ , which is  $\sim 11$  according to Gammie (1996). This is a large value indicating that galaxy gas disks should be destabilized by viscosity. An important dimensionless parameter for magnetic tension is  $B^2/(\pi G\Sigma^2) \sim 8$ , which is also large enough to be important. Thus gas disks should be generally unstable to form small spiral arms and clouds, even with moderately stable  $Q$ , although the growth rate can be low if  $Q$  is large.

## 1.2 Modern Versions of the KS Law with $\sim 1.5$ slope

Kennicutt et al. (2007) studied the local star formation law in M51 with 0.5-2 kpc resolution using Pa- $\alpha$  and  $24\mu$ +H $\alpha$  lines for the SFR, and a constant conversion factor for CO to H $_2$ . There was a correlation, mostly from the radial variation of both SFR and gas surface density, with a slope of  $1.56 \pm 0.04$ . There was no correlation with  $\Sigma(\text{HI})$  alone, as this atomic component had about constant column density ( $\sim 10 M_\odot$ ). The correlation with molecules alone was about the same as the total gas correlation.

Leroy et al. (2005) studied dwarf galaxies and found that they have a molecular KS index of  $1.3 \pm 0.1$ , indistinguishable from that of spirals, except with a continuation to lower central H $_2$  column densities (i.e., down to  $\sim 10 M_\odot \text{ pc}^{-2}$ ).

Heyer et al. (2004) found a slope  $n = 1.36$  for  $\Sigma_{\text{SFR}}$  versus  $\Sigma(\text{H}_2)$  in M33, where the molecular fraction,  $f_{\text{mol}}$  is small. The correlation with the total gas was much steeper. More recently, Verley et al. (2010) studied M33 again and got  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{H}_2}^n$  for  $n = 1$  to 2, and  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{total gas}}^n$  for  $n = 2$  to 4. The steepening for total gas is again because  $\Sigma_{\text{HI}}$  is about constant, so the slope from HI alone is nearly infinite. This correlation is dominated by the radial variations in both quantities, as it is a point-by-point evaluation throughout the disk. Radial changes in metallicity, spiral arm activation, tidal density, and so on, are part of the total correlation. Verley et al. (2010) also try other laws, such as  $\Sigma_{\text{SFR}} \propto (\Sigma_{\text{H}_2}\rho_{\text{ISM}}^{0.5})^n$ , for which  $n = 1.16 \pm 0.04$ , and  $\Sigma_{\text{SFR}} \propto \rho_{\text{ISM}}^n$ , for which  $n = 1.07 \pm 0.02$ . These differ by considering the conversion from column density to midplane density, using a derivation of the gaseous scale height. The first of these would have a slope of unity if the star formation rate per unit molecular gas mass were proportional to the dynamical rate at the average local (total) gas density. The second has the form of the original Schmidt law, which depends only on density. To remove possible effects of CO to H $_2$  conversion, Verley et al. also looked for a spatial correlation with the  $160 \mu$  opacity,  $\tau_{160}$ , which is a measure of the total gas column density independent of molecule formation. They found  $\Sigma_{\text{SFR}} \propto \tau_{160}^n$  for  $n = 1.13 \pm 0.02$ , although the correlation was not a single power law but a 2-component power law with a shallow part (slope  $\sim 0.5$ ) at low opacity ( $\tau_{160} < 10^{-4}$ ) and a steep part (slope  $\sim 2$ ) at high opacity.

### 1.3 Explanations for the 1.5 slope

Prior to around 2008, the popular form of the KS law had a slope of around 1.5 when  $\Sigma_{\text{SFR}}$  was plotted versus total gas column density on a log-log scale. This follows from a dynamical model of star formation in which the SFR per unit area equals the available gas mass per unit area multiplied by the rate at which this gas mass gets converted into stars, taken to be the dynamical rate,

$$\Sigma_{\text{SFR}} \sim \epsilon \Sigma_{\text{gas}} (G \rho_{\text{gas}})^{1/2}. \quad (1.2)$$

If the gas scale height is constant, then  $\Sigma_{\text{gas}} \propto \rho_{\text{gas}}$  and  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{1.5}$ . In the model of star formation where star-forming clouds are made by large-scale gravitational instabilities, this 1.5 power law would work only where the Toomre instability condition,  $Q \leq 1.4$ , is satisfied. Such a model accounts for the Kennicutt (1989, 1998) law with the  $Q < 1.4$  threshold.

Several computer simulations have shown this dynamical effect. Li et al. (2006) did SPH simulations of galaxy disks with self-gravity forming sink particles at densities larger than  $10^3 \text{ cm}^{-3}$ . They found a  $Q$  threshold for sink particle formation, and had a nice fit to the KS law with a slope of  $\sim 1.5$ . Tasker & Bryan (2006) ran ENZO, a 3D adaptive mesh code, with star formation at various efficiencies, various temperature floors in the cooling function, and various threshold densities. Some models had a low efficiency with a low threshold density and other models had a high efficiency with a high threshold density. Some of their models had feedback from young stars. They also got a KS slope of  $\sim 1.5$  for both global and local star formation, regardless of the details in the models. Kravtsov (2003) did cosmological simulations using N-body techniques in an Eulerian adaptive mesh. He assumed a constant efficiency of star formation at high gas density, and star formation only in the densest regions ( $n > 50 \text{ cm}^{-3}$ , the resolution limit), which are in the tail of the density probability distribution function (pdf; cf. Elmegreen, 2002; Krumholz & McKee, 2005). Kravtsov (2003) got the KS law with a slope of 1.4 for total gas surface density. Wada & Norman (2007) did a similar thing, using the fraction of the mass at a density greater than a critical value from the pdf ( $\rho_{\text{crit}} = 10^3 \text{ cm}^{-3}$ ) to determine the star formation rate. Their analytical result had a slope of 1.5. Harfst, Theis & Hensler (2006) had a code with a hierarchical tree for tracking interacting star particles, SPH for the diffuse gas, and sticky particles for the clouds. They included mass exchange by condensation and evaporation, mass exchange from stars to clouds (via PNe) and from stars to diffuse gas (SNe), and from clouds into stars during star formation. New clouds were formed in expanding shells. Their KS slope was  $1.7 \pm 0.1$ . They also got a drop in  $\Sigma_{\text{SFR}}$  at low  $\Sigma_{\text{gas}}$ , not from a  $Q$  threshold but from an inability of the gas to cool and form a thin disk (cf. Burkert et al., 1992; Elmegreen & Parravano, 1994).

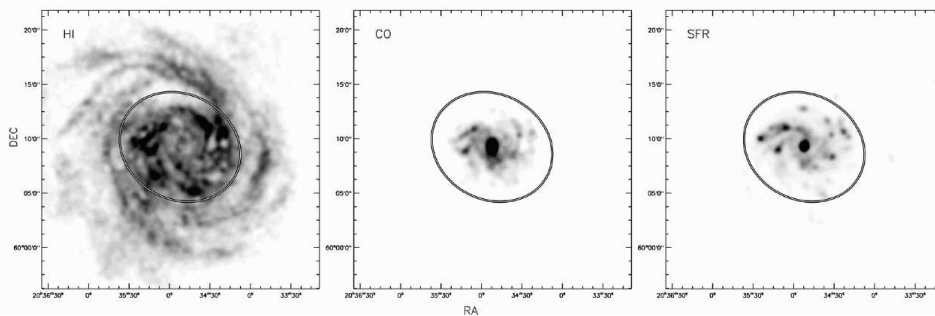
## 2 The Molecular Star Formation Law

The star formation law may also be written as a linear relation for molecules, with  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{H}_2}^1$  (e.g., Rownd & Young, 1999). Wong & Blitz (2002) found a SFR in

direct proportion to molecular cloud density ( $n = 1$ ), and suggested that the  $n = 1.4$  KS law came from changes in the molecular fraction,  $f_{\text{mol}} = \Sigma_{\text{H}_2} / (\Sigma_{\text{HI}} + \Sigma_{\text{H}_2})$ . They assumed that  $\text{H}_2/\text{CO}$  was constant and determined the combined index  $n' = n_{\text{mol}} (1 + d \ln f_{\text{mol}} / d \ln \Sigma_{\text{gas}})$  where  $n_{\text{mol}} = 1$  and  $f_{\text{mol}}$  increases with pressure,  $P$ . They measured  $d \ln f_{\text{mol}} / d \ln P \sim 0.2$ , and if  $P \propto \Sigma_{\text{gas}}^2$ , then  $d \ln f_{\text{mol}} / d \ln \Sigma_{\text{gas}} \sim 0.4$ . This gives the KS  $n = 1.4$  law for total gas. Wong & Blitz also suggested that the stability parameter  $Q$  was not a good threshold for star formation, but a better measure of the gas fraction in the sense that a high  $Q$  corresponds to a low  $\Sigma_{\text{gas}} / \Sigma_{\text{tot}}$ . Blitz & Rosolowski (2006) showed for a wider sample of 13 galaxies that the molecular ratio,  $R_{\text{mol}} = \Sigma_{\text{H}_2} / \Sigma_{\text{HI}}$ , scales about linearly with the total ISM pressure. Interacting galaxies had slightly higher  $R_{\text{mol}}$  for a given  $P$ , but among interacting galaxies, the correlation was still present.

A large study of HI, CO, and star formation rates from GALEX ultraviolet and Spitzer  $24\mu$  observations was made by Bigiel et al. (2008) and Leroy et al. (2008). They considered the local star formation law with a resolution of 750 pc. Bigiel et al. found that  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{CO}}$ , and that the timescale for conversion from  $\text{H}_2$  to stars was about 2 Gyr. Figure 1 (from Bigiel et al., 2008) shows an example of how much better the SFR scales with CO than either HI or the total gas. The CO and SFR maps of NGC 6946 resemble each other closely, and neither resembles the HI map. Bigiel et al. also found that  $\Sigma_{\text{HI}}$  saturates to  $\sim 9 M_{\odot} \text{ pc}^{-2}$ . When plotting  $\Sigma_{\text{SFR}}$  over a wide range of  $\Sigma_{\text{HI}+\text{H}_2}$ , they found a slope of unity in the molecular range,  $\Sigma_{\text{HI}+\text{H}_2} > 9 M_{\odot} \text{ pc}^{-2}$ , and higher slope in the atomic range ( $\Sigma_{\text{HI}+\text{H}_2} < 9 M_{\odot} \text{ pc}^{-2}$ ). Figure 2 shows the summed distribution of SFR per unit area versus total gas column density in 7 spiral galaxies. There is a linear part at high column density and a steeper part at low column density.

Dwarf galaxies look like the outer parts of spirals in the Bigiel et al. survey, occupying the steeper part of the  $\Sigma_{\text{SFR}} - \Sigma_{\text{gas}}$  diagram at low  $\Sigma_{\text{gas}}$ . At higher  $\Sigma_{\text{HI}+\text{H}_2}$ , the survey did not have new data, but Bigiel et al. suggested, based

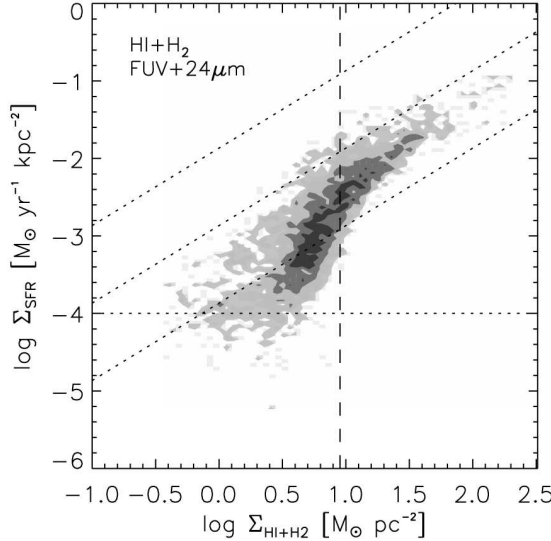


**Fig. 1.** Maps of HI, CO and SFR in NGC 6946 with HI on the left, CO in the middle and SFR on the right, all convolved to 750 pc resolution (from Bigiel et al. (2008)). The circle is the optical radius at 25 mag arcsec $^{-2}$ .

on Kennicutt’s (1998) starburst result, that perhaps the KS law turned up to a steeper slope ( $n \sim 1.4$ ) in a third regime of star formation where  $\Sigma_{\text{H}_2}$  exceeds the standard column density of a single molecular cloud (around  $100 M_\odot \text{ pc}^{-2}$ ).

Leroy et al. (2008) compared these new survey results to various theoretical models. They found that the star formation time in CO-rich gas is universally 1.9 Gyr, independent of the average local free fall or orbital time, the midplane gas pressure, the state of gravitational stability of the disk with or without the inclusion of stars in the stability condition, and regardless of the rate of shear or the ability of a cold gas phase to form. Star formation depends only on the presence of molecules and it proceeds at a fixed rate per molecule. Leroy et al. also found that dwarf galaxies are forming stars at their average historical rate, whereas spirals are forming stars at about half of their average rate. In the outer disk, the SFR in HI drops with radius faster than the free fall time, suggesting self-gravity is not the lone driver. Also important are the phase balance between HI and  $\text{H}_2$ , giant molecular cloud (GMC) destruction, stellar feedback, and other processes. These processes govern the presence of GMCs with an apparently constant star formation efficiency in each GMC.

Unlike the star formation rate per molecule, the molecule-to-atom ratio does



**Fig. 2.** The distribution of SFR per unit area versus total gas column density, convolved to 750 pc, for 7 spiral galaxies (from Bigiel et al. (2008)). There is a change in the slope from  $\sim 4$  at  $\Sigma_{\text{HI}+\text{H}_2} < 9 M_\odot \text{ pc}^{-2}$  (the vertical dashed line) in the outer disk to  $\sim 1$  at higher  $\Sigma_{\text{HI}+\text{H}_2}$  in the inner disk. The short-dashed lines correspond to gas depletion times of 0.1 Gyr, 1 Gyr, and 10 Gyr, from top to bottom.



correlate well with environmental parameters. Leroy et al. (2008) showed approximately linear correlations with stellar surface density and interstellar pressure, an inverse squared dependence on the orbit time, and an exponential dependence on the galactic radius, like the rest of the disk, and with a comparable radial scale length. The molecular fraction is a smooth function of environmental parameters (e.g., pressure); no thresholds were seen. Disks seem to be marginally stable throughout.

Leroy et al. concluded by noting that the HI-H<sub>2</sub> transition in spirals typically occurs at  $0.43 \pm 0.18 R_{25}$ , which is about the same as where  $\Sigma_{\text{stars}} = 81 \pm 25 M_{\odot} \text{ pc}^{-2}$ ,  $\Sigma_{\text{gas}} = 14 \pm 6 M_{\odot} \text{ pc}^{-2}$ ,  $P = 2.3 \pm 1.5 \times 10^4 k_{\text{B}} \text{ K cm}^{-3}$ , and  $T_{\text{orbit}} = 1.8 \pm 0.4 \text{ Gyr}$ . There should be unobserved H<sub>2</sub> in dwarfs, according to the high star formation rates and low CO emissions there; in fact Leroy et al. estimate for dwarfs  $\Sigma_{\text{H}_2} \sim 2\Sigma_{\text{HI}}$  in the inner regions.

Where  $\Sigma_{\text{HI}} > \Sigma_{\text{H}_2}$ , the star formation efficiency is proportional to  $\Sigma_{\text{stars}}$ , making

$$\Sigma_{\text{SFR}} \sim \Sigma_{\text{gas}} \times [\Sigma_{\text{stars}} / 81 M_{\odot} \text{ pc}^{-2}] / 1.9 \text{ Gyr}. \quad (2.1)$$

## 2.1 Theoretical Models for the Bigiel-Leroy Observations

Krumholz et al. (2008) considered the molecule formation problem by starting with the radiative transfer of H<sub>2</sub>-dissociating radiation:  $dF/dz = -n\sigma_{\text{d}}F - f_{\text{HI}}n^2R/f_{\text{diss}}$ . Here,  $F$  is the flux in Lyman-Werner bands that dissociate H<sub>2</sub>,  $n$  is the density ( $\sim 30 \text{ cm}^{-3}$  near the H<sub>2</sub> transition),  $\sigma_{\text{d}}$  is the dust cross section per H ( $10^{-21} \text{ cm}^2$ ),  $f_{\text{HI}}$  is the fraction of  $n$  that is HI,  $R$  is the rate coefficient for formation of H<sub>2</sub> on grains ( $\sim 3 \times 10^{-17} \text{ cm}^3 \text{ s}^{-1}$ ; i.e., the formation rate is  $f_{\text{HI}}n^2R$ ), and  $f_{\text{diss}}$  is the fraction of uv photon absorptions that dissociate H<sub>2</sub> ( $\sim 0.1$ ).

The solution to this radiative transfer equation is  $F(\tau) = (e^{-[\tau - \tau_{\text{HI}}]} - 1) / \chi$  where  $\tau_{\text{HI}} = \ln(1 + \chi)$ ;  $\chi = f_{\text{diss}}\sigma_{\text{d}}cE_0/nR$  is the ratio of absorption in dust to H<sub>2</sub>, and  $E_0$  is the free space photon number density ( $\sim 7.5 \times 10^{-4} \text{ cm}^{-3}$ ). Krumholz et al. (2008) assume that the cold neutral density comes from two phase equilibrium. Then  $n$  scales with  $E_0$  and  $\chi$  becomes nearly constant. From this they get the extinction,  $A_{\text{V}}$ , to the HI/H<sub>2</sub> transition, the HI column density,  $\Sigma_{\text{HI}}$ , and the molecular fraction in spherical cloud complexes as a function of the complex total column density. They do this also as a function of metallicity.

After considering a galactic cloud population, Krumholz et al. (2009a) derive  $\Sigma_{\text{HI}}$  versus  $\Sigma_{\text{total gas}}$  for different metallicities, and compare this with observations of galaxies having those metallicities. They do the same for H<sub>2</sub>. They also compare the observed versus the predicted correlation between H<sub>2</sub>/HI and pressure  $P$ . To do this, they use the observed  $\Sigma_{\text{total gas}}$  and metallicity, and then compute  $R_{\text{H}_2} = \Sigma_{\text{H}_2}/\Sigma_{\text{HI}}$  from theory. This is plotted versus the observed pressure from Blitz & Rosolowski (2006) and Leroy et al. (2008). The agreement is good.

Krumholz et al. (2009b) considered the star formation law,

$$\Sigma_{\text{SFR}} = \Sigma_{\text{gas}} f_{\text{H}_2} S F R_{\text{ff}} / t_{\text{ff}} \quad (2.2)$$

where the star formation rate in a free fall time is the fraction of the gas that turns into stars in a free fall time,  $SFR_{\text{ff}}$ , divided by the free fall time,  $t_{\text{ff}}$ . This is  $SFR_{\text{ff}}/\tau_{\text{ff}} = (M_6^{-0.33}/0.8 \text{ Gyr}) \times \text{Max}[1, \Sigma_{\text{gas}}/85 M_{\odot} \text{ pc}^{-2}]$ ;  $M_6$  is the cloud mass in units of  $10^6 M_{\odot}$ . This equation assumes that stars form in the high density tail of a log-normal density pdf, with the tail width given by the Mach number; a fraction of 0.3 of the dense gas mass goes into stars. The clouds are virialized and at uniform pressure until the galactic  $\Sigma_{\text{gas}}$  exceeds the column density of a single GMC; then the pressure equals the galactic pressure. Also, the cloud complex mass is taken to be  $M_6 = 37\Sigma_{\text{gas}}/(85 M_{\odot} \text{ pc}^{-2})$  from the galaxy Jeans mass.

This theory for molecule formation and star formation in a galactic environment fits well to the observations by Bigiel et al. (2008) and Leroy et al. (2008). It reproduces the low column density regime by having a low ratio of molecules to atoms at low pressure, it reproduces the intermediate column density regime by having a fixed star formation rate per molecule and an areal average star formation rate from the areal density of molecular clouds at constant pressure, and it reproduces the high column density regime by increasing the interstellar pressure, which makes the cloud density go up and the free fall time go down. A key point in their model is that molecular cloud pressures are constant in normal galaxy disks because they are set by HII region pressures (feedback) and not the galactic environment. In this sense, all GMCs have to be parts of shells or other active disturbances formed by high pressures.

We know that molecular cloud pressures in the Milky Way are about constant from the Larson (1981) laws, which require this for virialized clouds, but we don't really know the reason for it. It could be feedback, as Krumholz et al. (2009b) suggest, or it could be the weight of the HI shielding layer, which has a regulatory effect on pressure (Elmegreen, 1989). This regulatory effect works because at high ambient pressure, the atomic density on the periphery of molecular clouds is high and so the required surface column density for  $\text{H}_2$  line self-shielding is low, and vice versa. The pressure at the bottom of the shielding layer, which is the molecular cloud surface pressure, scales directly with the column density of the shielding layer. Thus a lower intercloud pressure is compensated by a higher HI column density at the molecular cloud surface, making the molecular cloud surface pressure somewhat uniform.

Robertson & Kravtsov (2008) simulated star formation in galaxies. They took a star formation rate per unit volume

$$d\rho_{\text{stars}}/dt = f_{\text{H}_2} \times (\rho_{\text{gas}}/t_{\text{SF}}) \times (n_{\text{H}}/[10 \text{ cm}^{-3}])^{0.5} \quad (2.3)$$

where  $t_{\text{SF}} = t_{\text{ff}}/\epsilon_{\text{ff}}$  is the free fall time,  $t_{\text{ff}}$ , divided by the fraction of the gas that turns into stars in a free fall time,  $\epsilon_{\text{ff}} = 0.02$ . To determine the molecular fraction,  $f_{\text{H}_2}$ , they considered heating and cooling, a radiation field proportional to the SFR, the Sternberg, et al. (2002)  $\text{H}_2$  formation theory, and radiative transfer using the *Cloudy* code. The result was a SFR that scaled steeply with the total gas column density, as observed, a higher KS slope for lower mass galaxies, which is also observed, and a shallower KS slope for the  $\text{H}_2$  column density alone, as in the molecular KS law. These results were somewhat independent of galaxy

mass. The molecular/atomic ratio also scaled with pressure in an approximately linear fashion, regardless of galaxy mass, as observed. They also found a stability parameter  $Q$  that ranged from unstable in the inner, star-forming parts of the disk, to stable in the outer regions.

## 2.2 Observations and Models of Outer Disks

Murante et al. (2010) have a multi-phase SPH code that assumes pressure determines the molecular abundance, and the molecules give the SFR. Below  $\Sigma_{\text{total gas}} \sim 10 M_{\odot} \text{ pc}^{-2}$ , the slope of the molecular star formation law turns out to be very steep,  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{total gas}}^n$  for  $n \sim 4$ . Above  $10 M_{\odot} \text{ pc}^{-2}$ , the slope is the same as in the Kennicutt law,  $n = 1.4$ , which is steeper than in Bigiel et al. (2008), where  $n \sim 1$  for the molecular Schmidt law.

Bush et al. (2010) simulated galactic star formation with special attention to the outer disks. The star formation model followed Springel & Hernquist (2002) with radiative cooling, star formation in the cold phase, no specific molecular phase, and a volume-Schmidt law,  $\rho_{\text{SFR}} \propto \rho_{\text{total gas}}^{1.5}$ . They found patchy star formation in the outer parts, usually along spiral arcs where the gas density was high. This morphology is in agreement with GALEX observations (Thilker et al., 2005; Gil de Paz et al., 2005). The Bigiel et al. and Leroy et al. observations were matched qualitatively in these outer parts too: below  $\Sigma_{\text{gas}} \sim 10 M_{\odot} \text{ pc}^{-2}$ , the slope  $n \sim 6$  to  $8$  was steeper than in the observations (which also plot SFR versus total gas in the outer regions). Then it was less steep at higher column density, with a slope of  $n \sim 1.4$ , which agrees with the Kennicutt (1998) slope for total gas.

Outer disks can be Toomre-stable on average because the gas and star column densities are very low. This is especially true for dwarf galaxies (Hunter et al., 1998). It might be that magnetic fields and viscosity destabilize outer disks, as discussed in Section 1.1, but in any case, outer disks appear to be much more stable than inner disks. More importantly, the gas in outer disks is often far from uniform and the use of an average column density for  $Q$  is questionable. Locally there can be islands of high column density where  $Q$  is small enough to be in the unstable region (van Zee et al., 1996). These islands have to be larger than the Jeans mass, which might be  $10^7 M_{\odot}$ . Spiral arms and large disturbances in pressure could make unstable regions like this. Dong et al. (2008) found unstable islands of star formation in the outer part of M83. In these regions, the star formation followed a steep KS law from point to point with a slope of about 1.4 (Dong et al., 2008).

Bigiel et al. (2010a) found that outer disk star formation seen by GALEX follows the HI very well in M83, with a uniform consumption time of 100 Gyr per atom beyond  $1.5R_{25}$ . The form of the star formation is mostly in spiral arms. Outer disk arms could be spiral waves radiating from the inner disk (Bertin & Amorisco, 2010).

Boissier et al. (2008) observed the galaxy-integrated KS law in low surface brightness galaxies, using a SFR from GALEX NUV observations. For a given total HI mass, the star formation rate was low by a factor of  $\sim 5$  compared to normal

spirals, but over the whole range, the total star formation rate scaled directly with the HI mass. This is not the same as saying that the star formation rate per unit area scales directly with the HI column density because the observations are spread out in a plot like this with big galaxies on one side and small galaxies on the other.

Bigiel et al. (2010b) studied SF rates in the far-outer disks of 17 spiral and 5 dwarf galaxies, where the gas is highly HI dominated. The SF laws compare well with those in dwarf galaxies. There is no obvious Q threshold. They suggest that the total SF Law has three components, the extreme outer disk component that is HI dominated, a transition region where the molecular fraction increases to near unity, the molecular region inside of that, and the starburst component, where the surface density is higher than that of a single GMC.

### 2.3 *Scaling relations inside individual clouds*

Krumholz & Tan (2007) showed that the conversion rate from gas to stars per unit free fall time is about constant inside clouds over a wide range of densities. This implies that the SFR per unit volume scales with the 1.5 power of density, with the first 1 in the power coming from the mass per unit volume, and the 0.5 in the power coming from the free fall rate. This is like a KS law, but for individual GMCs. There is also a threshold column density for star formation inside GMCs of around  $\sim 5-7$  mag in V (Johnstone et al., 2004; Kirk et al., 2006; Enoch et al., 2006; Jørgensen et al., 2007).

Chen et al. (2010) studied the KS relation for individual GMCs in the LMC. They measured the star formation rate from both HII regions and by direct counting of young stellar objects. For YSO counting, the rate per unit area inside a cloud approximately satisfies the total-gas Kennicutt relation with the same time scale per atom,  $\sim 1$  Gyr. For these regions,  $\Sigma_{\text{HI}+\text{H}_2} \sim 100 M_{\odot} \text{ pc}^{-2}$ , larger than in the main parts of galaxy disks. Chen et al. also found that the areal rate of star formation was much lower in the long molecular ridge south of 30 Doradus than in the GMCs. Presumably this ridge is not strongly self gravitating, even though it is CO-rich.

## 3 **Summary**

The empirical star formation law on kpc scales is essentially one where star formation follows CO-emitting molecular gas with a constant rate per molecule, and the ratio of molecular to atomic gas scales nearly directly with the ISM pressure (Bigiel et al., 2008; Leroy et al., 2008). The rate per molecule corresponds to a consumption time of molecular gas equal to about 2 Gyr. The place in a galaxy where the transition occurs between HI dominance in the outer part to H<sub>2</sub> dominance in the inner part is at a pressure of  $P = 2.3 \pm 1.5 \times 10^4 k_B \text{ K cm}^{-3}$ . There also tend to be characteristic gas and stellar column densities at this place, and a characteristic galactic orbit time for all of the galaxies observed. Beyond this

radius is the atomic-dominated outer disk. There, the SFR scales directly with  $\Sigma_{\text{HI}}$ , and the consumption time is about 100 Gyr.

Theoretical models of these empirical laws include the atomic-to-molecular transition in individual clouds and a sum over clouds to give the galactic scaling laws. Star formation occurs only in the densest parts of the clouds, as determined by a combination of turbulence-compression and self-gravity. Numerous simulations of star formation in galaxies can reproduce these empirical laws fairly well. The simulations usually show a sensitivity to the Toomre  $Q$  parameter, unlike the observations.

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